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Title: 'A plain linear rule for fatigue analysis under natural loading considering the sequence effect'

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Abstract

Fatigue under variable amplitude loading is currently assessed with the Palmgren-Miner rule in structural standards, ignoring the order of loading, which would require non-linear or mixed rules, especially for the random loading sequences applied to certain structures. Therefore, the goal is to develop a practical and simple correction factor ensuring the linear summation of damage is conservative, so as to take the sequence effect into account in random loading from natural sources. The theoretical consistency of this approach is demonstrated and a case study is developed to test the feasibility of the new rule and its simplicity.

Keywords: cumulative damage, variable amplitude fatigue, damage accumulation, sequence effect, random loading.

1 Introduction

It is well known in the literature that the sequential order of cycles is an important factor [1, 2, 3, 4, 5, 6, 7, 8] when assessing total cumulative fatigue damage. In fact, for certain sets of cycles, when larger ranges are applied beforehand, the resulting fatigue damage accumulation is higher, while precisely the opposite holds true in other cases [9, 10, 11, 12]: an aspect that will be explained in §2. However, simple linear approximations, such as the widely used Palmgren-Miner rule [13, 14], despite their practical and easy application, are unable to consider this effect [8, 15]. So, consideration of the sequence effect will require a more complex and therefore less practical non-linear rule [1, 2, 8, 15, 16]: as detailed in §3. Logically, the question arises of how to predict complex random loading from natural sources, i.e. wind, waves, seismic events, human-induced vibrations, etc., with disordered cycles of varying ranges, normally studied as stationary and ergodic processes. In such cases, the use of a non-linear rule for cumulative fatigue damage could be more accurate, but also more time intensive and of greater difficulty. In common structural elements and Eurocode design calculations [17, 18, 19] or equivalent structural standards, the real added value of a rule is its simplicity coupled with accuracy and safety [7]. These are the main advantages that justify why the Palmgren-Miner [13, 14] rule is still specified in these standards: although not more accurate, it is very practical, so the goal is to develop a practical correction factor ensuring the linear summation of damage is conservative. Following this acknowledged line of reasoning in engineering, a plain

linear-rule for cumulative fatigue damage, considering the sequence effect in complex random loading, is presented in §4. An accelerating coefficient is presented for this process [20], analogous to the approach for the “damage equivalent factor” concept [7]. Finally, the practical application of this rule is presented in the case study of a steel rod in §5. Besides, for a wider review of existing fatigue rules see [5, 6, 20].

2 The sequence effect in cumulative fatigue damage

Now, a short introduction on this topic looking for a triple benefit: First, to see how the sequence order is currently taken into account, even in a rudimentary way, for cumulative fatigue damage under a linear rule, to state the starting point. Second, how the uncertainty on the order could change the curve itself below the CAFL and why is that. Finally, third, the reason why the Hi-Lo sequences are more damaging than the Lo-Hi, related with crack size.

Sequential cycles in a fatigue process can shorten the service life of certain structural elements [1, 2, 3, 4, 5, 6, 7, 8], in terms of the number of cycles an element will resist until failure; a process that can be understood from the perspective of fracture mechanics. If a low cyclic stress range under a given threshold stress intensity factor is applied before any other to an initial crack size, crack propagation will not occur. However, the application in the first place of a high cyclic stress range will propagate an increase in the initial crack size. Thus, the new crack size will reduce the threshold stress intensity factor, causing further crack propagation under subsequent lower stress ranges, so cumulative fatigue damage will therefore occur under both stress ranges, instead of only the higher ones in the opposite case [8].

This effect can be observed in the S-N or the $\Delta\sigma$ -N curves [21], which reflect the two stress range levels defined for completely reversed cycles. A first one called the fatigue endurance limit or Constant Amplitude Fatigue Limit (CAFL) and a second one, lower than the first, called the cut-off limit [7, 17, 18, 19]. The curves horizontally flatten in log-log scale, under the fatigue limit, if all cycles in the sequence are of a lower stress range. However, if there are some cycles with higher stress levels, the slope is intermediate, representing the availability of lower stress range cycles to produce fatigue damage after some higher stress range. However, there is a cut-off limit that represents the point where fatigue damage is inexistent, because the cyclic loading under this limit will not result in crack growth.

Finally, the sequence effect in the fatigue processes has been studied in the literature and empirically verified several times [9, 10, 11, 12], pointing to the acceleration of these processes in cyclic blocks, ordered by decreasing stress ranges and the corresponding deceleration in the opposite case. The Palmgren-Miner [13, 14] linear rule is unable to predict these effects that are introduced by the commutative property of the summation of the partial cumulative fatigue damage of each block.

3 Non-linear rule approach

In this section, the sequence effect on complex random loading will be studied using a non-linear rule, see [22, 1, 2, 5, 8, 15, 16]. First, cumulative fatigue damage D is expressed in a general equation (1), where a certain cycle block i with a constant stress range $\Delta\sigma_i$ has an exponent $\omega_i=f(\Delta\sigma_i)$, anything other than 1 and higher the lower

is the stress range. Finally, the number of cycles in that block i is n_i and the number of cycles until failure at stress range $\Delta\sigma_i$ is Nf_i

$$D = \sum \left(\frac{n_i}{Nf_i} \right)^{\omega_i} \quad (1)$$

Thus, the first step to calculate the total damage D_T caused by two consecutive blocks of cycles is to derive the damage of the first block D_1 , see (2).

$$D_1 = \left(\frac{n_1}{N_{f1}} \right)^{\omega_1} \quad (2)$$

Then, the same with the damage of the second block D_2 . However, this time it is more complex, since the element is not new and has been previously fatigued, so block 2 starts with an initial damage $D_{0,2}$. Logically, the damage with which the second block starts is precisely the damage with which the first one ends, so $D_{0,2}=D_1$. Besides, to continue with damage progression according to second block damage curve, the initial damage $D_{0,2}$ should be expressed in terms of the equivalent initial second block cycles $n_{0,2}$, see (6).

$$D_{0,2} = D_1 \rightarrow \left(\frac{n_{0,2}}{N_{f2}} \right)^{\omega_2} = \left(\frac{n_1}{N_{f1}} \right)^{\omega_1} \quad (3)$$

Therefore, the total damage $D_T=D_1+D_2$, expressed in terms of cycles from the second block curve cycles n_2 , is derived in (4).

$$D_T = \left(\left(\frac{n_{0,2}}{N_{f2}} \right) + \left(\frac{n_2}{N_{f2}} \right) \right)^{\omega_2} \quad (4)$$

Besides, substituting equation (3) into equation (4), then the total damage $D_T=D_1+D_2$ could be expressed in terms of cycles in both blocks, n_1 and n_2 , see (5).

$$D_T = \left(\left(\frac{n_1}{N_{f1}} \right)^{\frac{\omega_1}{\omega_2}} + \left(\frac{n_2}{N_{f2}} \right) \right)^{\omega_2} \quad (5)$$

Finally, the damage specifically done during second block D_2 can be derived subtracting from the total damage D_T , expressed in equation (5), the damage done during the first one D_1 , expressed in equation (2), yielding equation (6).

$$D_2 = \left(\left(\frac{n_{0,2}}{N_{f2}} \right) + \left(\frac{n_2}{N_{f2}} \right) \right)^{\omega_2} - \left(\frac{n_1}{N_{f1}} \right)^{\omega_1} = \left(\left(\frac{n_1}{N_{f1}} \right)^{\frac{\omega_1}{\omega_2}} + \left(\frac{n_2}{N_{f2}} \right) \right)^{\omega_2} - \left(\frac{n_1}{N_{f1}} \right)^{\omega_1} \quad (6)$$

Moreover, in the case of a third block of cycles, the complexity of the operation will increase even further, as in (7) that shows the damage, D_3 , caused by this third block of cycles. This complexity will be increased after each new block of a different amplitude, becoming progressively less practical.

$$D_3 = \left(\left(\frac{n_{0,3}}{N_{f3}} \right) + \left(\frac{n_3}{N_{f3}} \right) \right)^{\omega_3} - D_1 - D_2 = \left(\left(\left(\frac{n_1}{N_{f1}} \right)^{\frac{\omega_1}{\omega_2}} + \left(\frac{n_2}{N_{f2}} \right)^{\frac{\omega_2}{\omega_3}} + \left(\frac{n_3}{N_{f3}} \right) \right)^{\omega_3} - \left(\left(\frac{n_1}{N_{f1}} \right)^{\frac{\omega_1}{\omega_2}} + \left(\frac{n_2}{N_{f2}} \right) \right)^{\omega_2} \right) \quad (7)$$

An interesting fact may be noted when analyzing these equations: there is an increment in terms of relative cycles, n/N_i , when passing from a curve to the next. When the fatigue process passes from the first block to the second block of cycles, it moves from corresponding damage curve 1 of block 1 to damage curve 2 of block 2, retaining the same level of damage, but there is a difference between the final quotient n_1/N_{f1} of the first, and the initial quotient $n_{0,2}/N_{f2}$ of the second; see (8).

$$\Delta \left(\frac{n}{N_f} \right)_{1 \rightarrow 2} = \left(\frac{n_{0,2}}{N_{f2}} \right) - \left(\frac{n_1}{N_{f1}} \right) = \left(\frac{n_1}{N_{f1}} \right)^{\frac{\omega_1}{\omega_2}} - \left(\frac{n_1}{N_{f1}} \right) \quad (8)$$

Analyzing equation (8), if the tendency of the non-linear rule were linear, then both ω_1 and ω_2 would also tend to be equal to 1, yielding no increment. Moreover, if the non-linear rule had a constant exponent, independent of the stress amplitude that corresponds to each block, then the increment would also be nil. A final important point is that, if the process moves from a block of cycles 1 with a certain stress amplitude to a block of cycles 2 with a lower stress amplitude, then the exponent, ω_1 , is lower than ω_2 , which implies a positive increase in terms of the relative quotient n/N_i of the cycle, thereby accelerating the cumulative fatigue damage that is approaching failure, while the opposite is verified when the first block has a lower stress amplitude than the second block of cycles; see (9).

$$\omega_1 < \omega_2 \rightarrow \frac{\omega_1}{\omega_2} < 1 \wedge \left(\frac{n_1}{N_{f1}} \right) < 1 \rightarrow \left(\frac{n_1}{N_{f1}} \right)^{\frac{\omega_1}{\omega_2}} > \left(\frac{n_1}{N_{f1}} \right) \rightarrow \Delta \left(\frac{n}{N_f} \right)_{1 \rightarrow 2} > 0 \quad (9)$$

Fig. 1 shows the graph of a fatigue process after four blocks of cycles of different stress amplitude, ordered by decreasing stress amplitude, up until failure. The graph represents the Palmgren-Miner cumulative fatigue damage curves [13, 14] with a dotted line and the others correspond to the stress amplitudes of each block. The X (ordinate) axis or abscissa represents the relative cycle quotient, or life fraction, n/N_i , and the Y (coordinate) axis represents the accumulated fatigue damage, D ; see [5, 8, 23]

The fatigue process represented in this graph has been quasi-linearized, showing horizontal lines that correspond to each cycle increment when moving from one curve to the next at the same damage level, and other positive slope lines, corresponding to the cumulative damage in each block of cycles. In terms of the abscissa coordinates, the summation of the relative cycle quotient of each block of cycles is equal to the result obtained by application of Palmgren-Miner rule [13, 14], while the difference between that result and 1 is the sum of the cycle increment caused by the sequence effect, when moving from one curve to the next. It is now evident that, when a fatigue process has positive cycle increments, the Palmgren-Miner rule [13, 14] will not predict failure when it indeed occurs and is therefore less accurate and potentially unsafe.

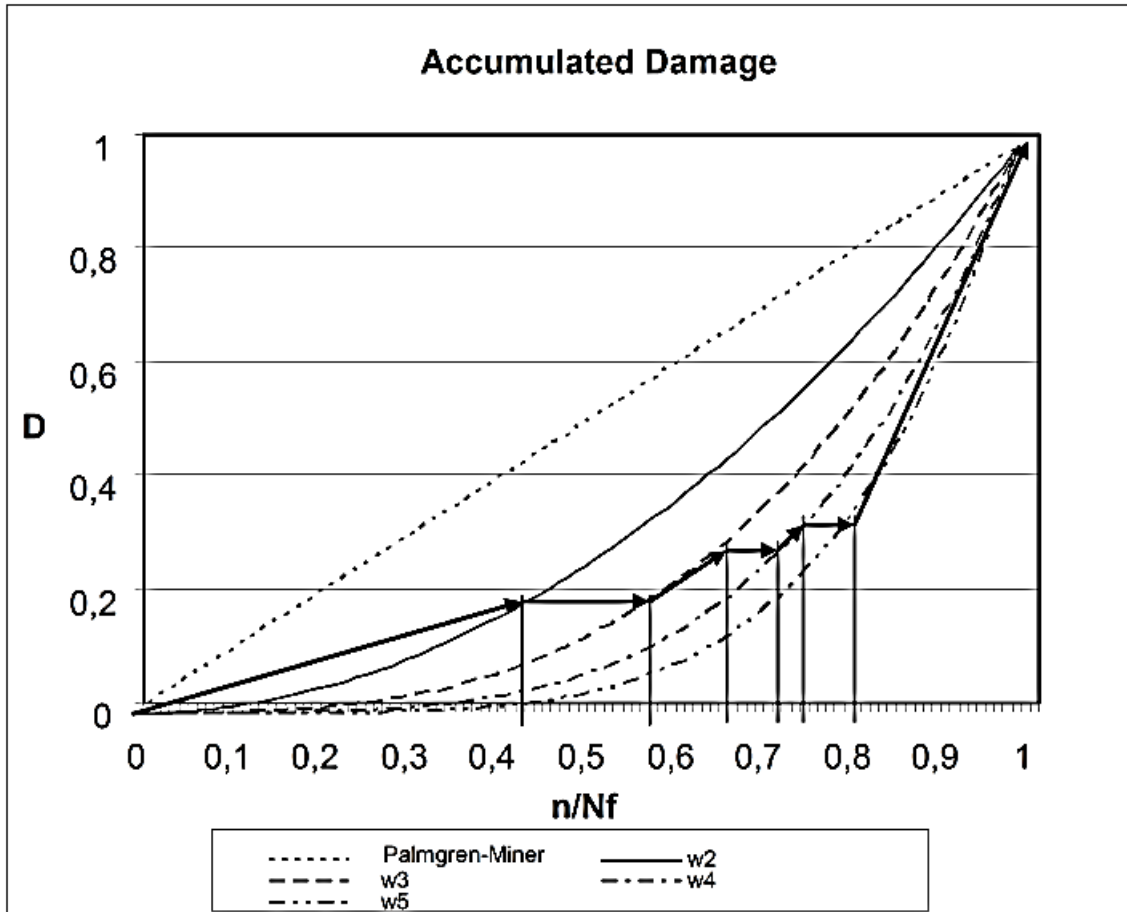


Fig. 1: Process of cumulative fatigue damage following 4 blocks of cycles ordered by decreasing stress amplitude [20].

Thus, in view of the above graph, the Palmgren-Miner [13, 14] rule may still be applied for certain load sequences, but multiplied by a coefficient of acceleration that pushes the fatigue process representing the loading sequence effect. The coefficient is equal to the inverse of 1 minus the sum of the expected relative cycle increments. This method is analogous to the “damage equivalent factor” concept approach used in the Eurocodes [7].

However, prediction of the sum of each cycle increment would be difficult in complex random loading of data from natural sources, because the sequence is very messy and disordered, meaning that it would not be, in principle, a very practical approach. Nevertheless, it is indeed possible to estimate the maximum expected value of the relative cycle increment and, therefore, the upper boundary of this coefficient. This approach reflects the dual advantage of the Palmgren-Miner rule [13, 14]: both practical and conservative.

4 Plain linear rule approach

Seeking to maintain practical, feasible and conservative fatigue damage predictions of elements under complex random loading from natural sources, the best approach is therefore to multiply the result of the calculation with a linear rule by an acceleration factor that takes into account the maximum expected relative cycle increment for any possible load sequence. We will call this factor a disorder pushing factor, as it actually

takes into account an upper boundary of the disorder effect of the sequence and it pushes the cumulative fatigue damage of the process, which is finally responsible for accelerating the failure of the structural element.

Therefore, the key to this approach is neither the identification nor the prediction of the actual loading sequence effect, which are tedious and time-consuming tasks of limited and partial accuracy. Instead, the key is to identify, for a certain load cycle sequence, the worst possible case in terms of the sequence effect, before the number of cycles is even counted and before all other considerations. This approach will guarantee that the disorder pushing factor will be the envelope value of any cyclic loading sequence with the same global maximum and minimum stress ranges.

This maximum relative cycle increment between two curves is obtained when changing, or moving from the first curve to the second precisely at the damage level where the distance between both curves is maximized. This point will be better explained in section 4.1. Repeating this move at the proper damage level between every consecutive pair of curves will configure the worst path, defined in 4.2, determining the cumulative fatigue process that maximizes the sum of relative cyclic increments between each pair of curves.

4.1 Maximum relative cycle increment

The relative cycle increment, caused when passing from a cycle block of greater amplitude to another of lesser amplitude, is shown in Fig. 2, in terms of n/N_f , as an isolated movement between the two consecutive curves corresponding to both blocks of cycles.

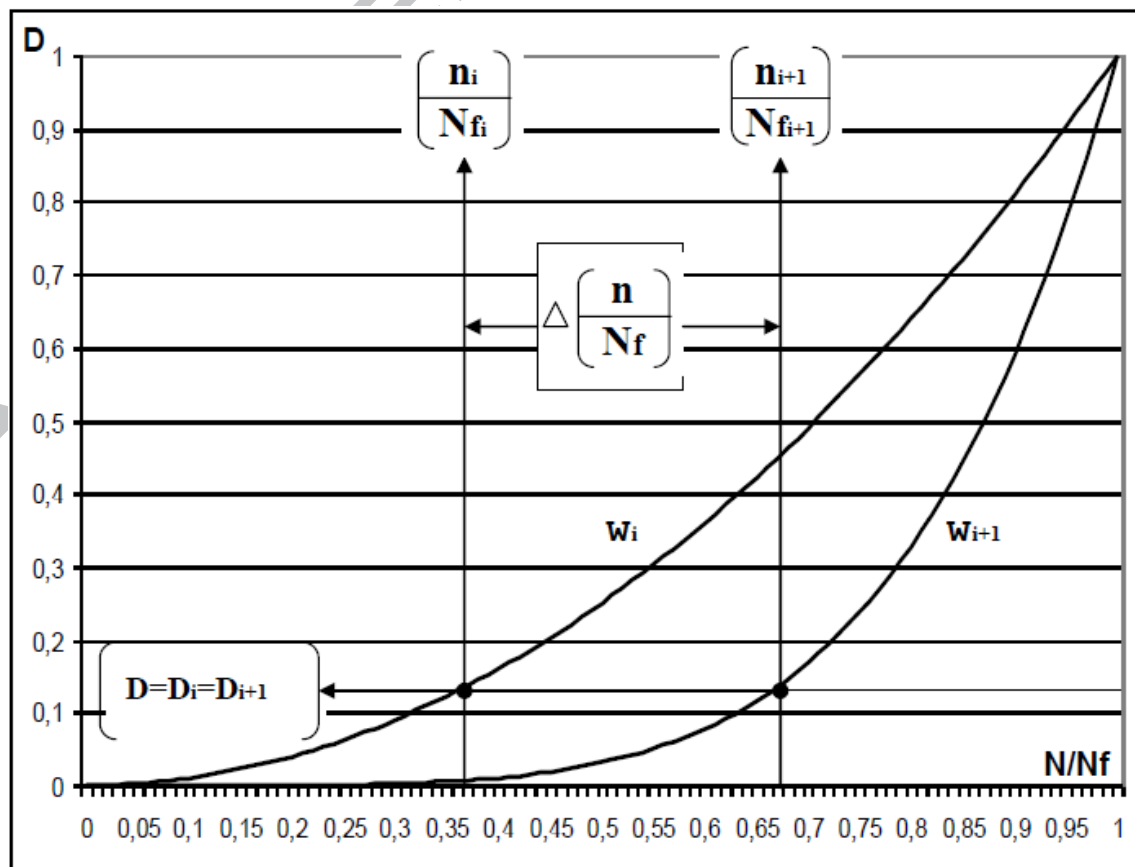


Fig. 2: Relative cycle increment when passing from one block of cycles of a higher range to another of a lower range [20].

For a certain set of blocks ordered by stress range, such that $\Delta\sigma_i > \Delta\sigma_{i+1}$, there are always two consecutive blocks of cycles, the corresponding cumulative fatigue damage curves of which are respectively defined by equations (10) and (11)

$$D_i = \left(\frac{n_i}{Nf_i} \right)^{\omega_i} \quad (10)$$

$$D_{i+1} = \left(\frac{n_{i+1}}{Nf_{i+1}} \right)^{\omega_{i+1}} \quad (11)$$

Now, considering that the movement between curves is always repeated, keeping the same damage level, $D_i = D_{i+1}$, then equation (12) is verified and, consequently, the corresponding relative cycle increment is defined by equation (13):

$$\left(\frac{n_{i+1}}{Nf_{i+1}} \right) = \left(\frac{n_i}{Nf_i} \right)^{\omega_i / \omega_{i+1}} \quad (12)$$

$$\Delta \left(\frac{n}{Nf} \right) = \frac{n_{0,i+1}}{Nf_{i+1}} - \frac{n_i}{Nf_i} = \left(\frac{n_i}{Nf_i} \right)^{\omega_i / \omega_{i+1}} - \frac{n_i}{Nf_i} \quad (13)$$

Thus, the maximum cycle increment can be determined as a derivative of equation (13), see equation (14), and by finding the relative cycle amount where it is nullified. Following this procedure, the relative cycles of the first curve (15) and the cumulative fatigue damage level (16) are addressed.

$$\frac{d \left[\Delta \left(\frac{n}{Nf} \right) \right]}{d \left(\frac{n_i}{Nf_i} \right)} = \frac{\omega_i}{\omega_{i+1}} \cdot \left(\frac{n_i}{Nf_i} \right)^{\frac{\omega_i - \omega_{i+1}}{\omega_{i+1}}} - 1 = 0 \quad (14)$$

$$\frac{n_i}{Nf_i} = \left(\frac{\omega_{i+1}}{\omega_i} \right)^{\omega_i / (\omega_i - \omega_{i+1})} = \left(\frac{\omega_i}{\omega_{i+1}} \right)^{\omega_{i+1} / (\omega_{i+1} - \omega_i)} \quad (15)$$

$$D = D_i = D_{i+1} = \left(\frac{n_i}{Nf_i} \right)^{\omega_i} = \left(\frac{\omega_i}{\omega_{i+1}} \right)^{\omega_i \cdot \omega_{i+1} / (\omega_{i+1} - \omega_i)} \quad (16)$$

Therefore, by substituting equation (15) into equation (13), the maximum relative cycle increment is determined in (17)

$$\Delta \left(\frac{n}{Nf} \right)_{\max} = \left(\frac{\omega_i}{\omega_{i+1}} \right)^{\omega_i / (\omega_{i+1} - \omega_i)} \cdot \left(1 - \frac{\omega_i}{\omega_{i+1}} \right) \quad (17)$$

Finally, two worthwhile observations may be noted. First, in view of (16), it is derived that the damage level at which the relative cycle increment is maximized decreases as

the relation ω_{i+1}/ω_i increases. Second, the movement between curves with a maximum relative cycle increment occurs between two points, one on each curve, that share the same damage level and derivative, that is, a tangential slope. Indeed, to demonstrate the preceding observation, it is evident that equation (18) could simultaneously represent the inverse of equations (10) and (11), the derivative of which is equation (19).

$$\left(\frac{n}{Nf}\right) = D^{1/\omega} \quad (18)$$

$$\frac{d\left(\frac{n}{Nf}\right)}{dD} = \frac{1}{\omega} \cdot D^{\frac{1-\omega}{\omega}} \quad (19)$$

Equalling out the derivatives of equations (10) and (11), both expressed in the form of equation (19), yields equation (20). But, considering that the increment is done at the same level of fatigue damage, i.e. $D_i = D_{i+1} = D$, then equation (20) can be developed into equation (21), which is equal to (16). This development of the equation demonstrates the second and last observation.

$$\frac{1}{\omega_i} D_i^{\frac{1-\omega_i}{\omega_i}} = \frac{1}{\omega_{i+1}} D_{i+1}^{\frac{1-\omega_{i+1}}{\omega_{i+1}}} \quad (20)$$

$$D = \left(\frac{\omega_i}{\omega_{i+1}}\right)^{\frac{\omega_{i+1} \cdot \omega_i}{\omega_{i+1} - \omega_i}} \quad (21)$$

4.2 Worst Path

In practice, a random loading will present several cycles in a certain order that is unpredictable. Thus, being unpredictable the order, so it is the exact sequence effect. However, if not able to derive the exact sequence effect, at least it could be possible to derive its maximum, as an upper boundary value to work with. If the damage calculation considering this maximum value was safe, so it was any less damaging sequence. In order to do that, all cycles gathered in blocks of similar stress range will be applied in the most damaging order. Therefore, the itinerary passing from a block to the next having the maximum relative cycle increment will be the worst path.

On this premise, the objective is now to extend the relation shown in section 4.1 to multiple blocks of cycles, at least over two, which reflect a generic sequence effect. The same study is followed for this task, but extending the same method to three consecutive blocks. These blocks are ordered and numbered by decreasing stress amplitudes. Thus, the initial stress range condition (22) is verified and, regarding the exponents of their corresponding cumulative fatigue damage curves, its immediate consequence (23).

$$\Delta\sigma_1 > \Delta\sigma_2 > \Delta\sigma_3 \quad (22)$$

$$1 < \omega_1 < \omega_2 < \omega_3 \quad (23)$$

For instance, if the derivative of equation (21) with respect to ω_{i+1} , with ω_i remaining constant, is always negative, it necessarily implies that the result of equation (21) will be lower while ω_{i+1} will increase and vice versa. Therefore, whatever the values of these exponents, it effectively implies a relation of $D_{13} < D_{12}$.

$$\frac{dD}{d\omega_{i+1}} = \left[\frac{-\omega_i^2}{(\omega_{i+1} - \omega_i)^2} \cdot \ln\left(\frac{\omega_i}{\omega_{i+1}}\right) - \frac{1}{\omega_{i+1}} \cdot \frac{\omega_i \cdot \omega_{i+1}}{\omega_{i+1} - \omega_i} \right] \left(\frac{\omega_i}{\omega_{i+1}} \right)^{\frac{\omega_{i+1} \cdot \omega_i}{\omega_{i+1} - \omega_i}} \quad (24)$$

Thus, if equation (24) is negative, regardless of the exponent values, the terms of the equation between square brackets will necessarily be negative, as the remainder is always positive. This condition is expressed in equation (25), which is further developed into equation (26).

$$\frac{dD}{d\omega_{i+1}} < 0 \rightarrow \frac{\omega_i^2}{(\omega_{i+1} - \omega_i)^2} \cdot \ln\left(\frac{\omega_{i+1}}{\omega_i}\right) < \frac{1}{\omega_{i+1}} \cdot \frac{\omega_i \cdot \omega_{i+1}}{\omega_{i+1} - \omega_i} \quad (25)$$

$$\ln\left(\frac{\omega_{i+1}}{\omega_i}\right) < \frac{\omega_{i+1} - \omega_i}{\omega_i} \quad (26)$$

Now, equation (26) is transformed into equation (28) by substitution of the key quotient expressed in equation (27), at all times greater than 1. Then, equation (29) is developed from (28), in which $c > 1$ is always the case and may always be verified with the corresponding limit that tends towards the asymptote.

$$c = \frac{\omega_{i+1}}{\omega_i} > 1 \quad (27)$$

$$\ln(c) + 1 - c < 0 \quad (28)$$

$$c \cdot e^{1-c} < 1 \quad (29)$$

Analogously, following the same procedure but keeping ω_{i+1} constant while obtaining the derivative with respect to ω_i , is developed the derivative (30). This derivative cannot itself be negative, unless the term between square brackets is also negative, which is expressed in (31), and subsequently operated upon in (32). The same substitution yields (33), which is developed into (34), finally resulting in (35), which is also verified for any value greater than 1. Finally, this implies that $D_{23} < D_{13}$, regardless of the exponent values while the relationship is maintained (23).

$$\frac{dD}{d\omega_i} = \left[\frac{\omega_{i+1}^2}{(\omega_{i+1} - \omega_i)^2} \cdot \ln\left(\frac{\omega_i}{\omega_{i+1}}\right) + \frac{1}{\omega_i} \cdot \frac{\omega_i \cdot \omega_{i+1}}{\omega_{i+1} - \omega_i} \right] \left(\frac{\omega_i}{\omega_{i+1}} \right)^{\frac{\omega_{i+1} \cdot \omega_i}{\omega_{i+1} - \omega_i}} \quad (30)$$

$$\frac{dD}{d\omega_i} < 0 \rightarrow \frac{1}{\omega_i} \cdot \frac{\omega_i \cdot \omega_{i+1}}{\omega_{i+1} - \omega_i} < \frac{\omega_{i+1}^2}{(\omega_{i+1} - \omega_i)^2} \cdot \ln\left(\frac{\omega_{i+1}}{\omega_i}\right) \quad (31)$$

$$\frac{\omega_{i+1} - \omega_i}{\omega_{i+1}} < \ln\left(\frac{\omega_{i+1}}{\omega_i}\right) \quad (32)$$

$$\frac{c-1}{c} < \ln(c) \quad (33)$$

$$0 < \ln(c) + \frac{1-c}{c} \quad (34)$$

$$1 < c \cdot e^{\frac{1-c}{c}} \quad (35)$$

Finally, as a result of these derivatives, on the one hand, if (29) is true for any value of 'a' greater than 1, then $D_{13} < D_{12}$ and, on the other hand, if (35) is true for any value of 'a', then $D_{23} < D_{13}$. This result is summarized in equation (36).

$$D_{23} < D_{13} < D_{12} \quad (36)$$

The consequences of this very last relationship are explained with the help of Fig. 3, which presents six cases with segments of the non-linear cumulative damage curves, depending on the relative cycles of the three consecutive blocks of cycles the exponents of which are ω_1 , ω_2 and ω_3 .

First, considering a), the movement with the maximum relative cycle increment is the one that passes from curve 1 to curve 3, the amount of which in terms of relative cycles is J_{13} . This movement occurs at a certain cumulative fatigue damage level, marked by the horizontal line. However, considering b), the point with the maximum relative increment J_{23} moving from curve 2 to curve 3, occurs at a lower level of fatigue damage; see (36). Finally, considering c), the movement from curve 1 to curve 2 has to be done before moving from 2 to 3, but the maximum relative cycle increment of moving from 1 to 2 J_{12} happens at a higher level of fatigue damage. Therefore, the maximum relative cycle increment occurs when moving from curve 1 to 2 at precisely the same damage level, since the relative cycle increment J_{12} will decrease if it is done any earlier. Now, considering c), the dotted line corresponds to J_{13} , that has been defined as the maximum relative cycle increment moving directly from curve 1 to curve 3, so in this case the maximum relative cycle increment is reached by passing directly from curve 1 to curve 3, since $J_{13} > J_{12} + J_{23}$ is always true.

Second, considering d), the maximum movement J_{13} passing from curve 1 to curve 3 happens at a certain damage level. Considering e), and (36), the maximum movement passing from curve 1 to curve 2 happens at higher level of fatigue damage. Since the movement from curve 2 to curve 3 has to be done after moving from curve 1 to curve 2, but the maximum movement from curve 2 to curve 3 is done at a lower level of fatigue damage, then the maximum relative cycle increment is reached if moving at precisely the same level, because if done later the increment will be reduced. Therefore, looking at f) it becomes true that the sum of J_{12} and J_{23} will be less than J_{13} , represented by the dotted line. So, in this case too, the maximum relative cyclic increment is reached by passing directly from curve 1 to curve 3, since $J_{13} > J_{12} + J_{23}$ is always true.

Therefore, the consequence of these two (36) issues is that every triple set of consecutive curves, where the relationships (22) and (23) are true, have a maximum relative cycle increment equivalent to the move from the first to the last. But, the real issue is raised here, as it could be extended for every fatigue process, of any number of blocks of any number of individual cycles, ordered in decreasing amplitude, which is the most damaging possible sequence in random loading. So, if the maximum relative cycle increment passing from curve 1 to curve 2, and then to curve 3, is to move directly from 1 to 3, then the same can be said for the next triple set of curves 1, 3, and

4, and the maximum increment will move directly from 1 to 4, so the same can be said of the triple set of curves 1, 4 and 5, and so on until the last triple set 1, N-1 and N, the maximum relative cycle increment of which, according to (17), but now expressed in terms of the loading sequence of L blocks of cycles that is defined in equation (37).

$$\Delta\left(\frac{n}{Nf}\right)_{\max} = \left(\frac{\omega_1}{\omega_L}\right)^{\frac{\omega_1}{\omega_N - \omega_1}} \cdot \left(1 - \frac{\omega_1}{\omega_L}\right) \quad (37)$$

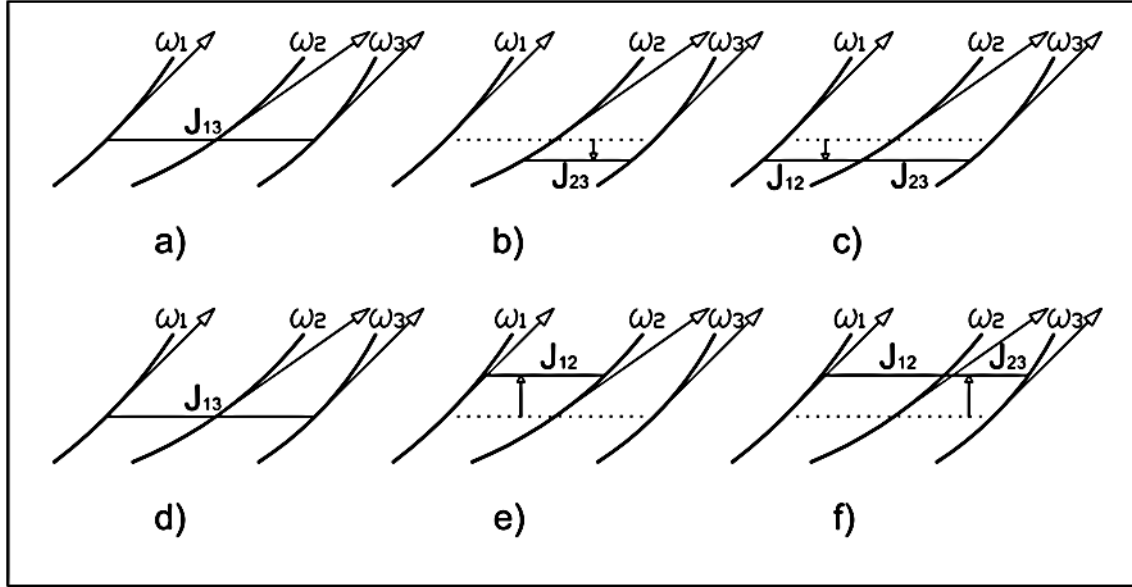


Fig. 3: Interpretation of consequences derived from the relationship expressed in equation (36), [20].

4.3 Disorder Pushing Factor

Finally, the disorder pushing factor F_{ED} , calculated in a general way for a certain set of blocks of cycles, will be valid for any order of the sequence, as it uses the maximum envelope value of the relative cycle increment. Besides, taking into account that the maximum relative cycle increment is the movement between the first and last cumulative fatigue damage curves, it will also be valid for any set of blocks of cycles sharing the maximum and minimum stress amplitude.

As a comment on conservatism of this rule for a stationary and ergodic random loading coming from a natural source, mind that there are too concomitant permanent loading and some time-dependent processes like creep, relaxation, shrinkage, inertia loose, bolt untightening, etc. that tend to redistribute stress within a structure, reducing the stress in a certain detail. The consequence is higher mean stresses at beginning of fatigue process than at the end, and corresponding higher equivalent stress ranges before. This induces a quasi-order in the stationary ergodic process in terms of equivalent cycles that is decreasing order.

Once the maximum relative cycle increment has been determined, corresponding to the worst path, the failure of the structural element by fatigue will occur when equation (38) is fulfilled. So, the factor by which the result of the Palmgren-Miner rule [13, 14] should be multiplied, which is the summation of only the relative cycles of each block of

cycles, should (39) predict the point of failure. Finally, substituting equation (37) in (39) yields equation (40), which is the general form of the factor. In this very last equation, 1 is the notation that indicates the first block of cycles, or the one with higher stress amplitude, and L is the notation to indicate the last block of cycles, or the one with the lowest level of stress amplitude in the sequence.

$$\sum \left(\frac{n}{Nf} \right) + \Delta \left(\frac{n}{Nf} \right)_{\max} = 1 \quad (38)$$

$$F_{ED} = \frac{1}{1 - \Delta \left(\frac{n}{Nf} \right)_{\max}} \quad (39)$$

$$F_{ED} = \frac{1}{1 - \left(\frac{\omega_1}{\omega_L} \right)^{\omega_1 / (\omega_N - \omega_1)} \cdot \left(1 - \frac{\omega_1}{\omega_L} \right)} \quad (40)$$

Therefore, the fatigue damage of a certain random fatigue load sequence can be analyzed by means of a linear rule, completing the corresponding histogram and applying the Palmgren-Miner procedure [13, 14], but multiplying the result by this disorder pushing factor that accelerates the deterioration process. The disordered linear rule therefore corresponds to equation (41), useful for easy safety limit calculations, a central added value, similar to those prescribed in structural standards.

$$D = F_{ED} \cdot \sum \left(\frac{n_i}{Nf_i} \right) \quad (41)$$

As a final remark, the proposed rule develops a correction factor to take into account sequence effect in random loadings under linear rule, which is the rule used in practice. The non-linear basis to derive this factor is consistent with fracture mechanics theory, since its exponent is found by means of it, as shown in next section. However, its added value is not to be more accurate with experimental data, inherently scattered. In fact, its truly added value is to keep simple while enabling the consideration of sequence effects, dealing with uncertainty in random loadings, that introduces double scattering, even being little conservative. For instance, a cycle by cycle approach like Mesmacque et al. [24] or others [25, 26] could be very accurate but it deals with certainty, i.e. with known sequences, such as those applied in laboratories. When the sequence is no longer previously known, like in random loadings, then uncertainty is introduced in the prediction and it becomes blurry and inaccurate, if not unable to be done. In such cases, where the accurate prediction is not possible, the next best option is to determine the safe prediction.

5. Case study: a steel rod

A steel rod is proposed as a case study [27], for the purpose of demonstrating the feasibility of this method. The principal advantage is that this geometry is easily extensible to elements such as rebars, bolts, cables, rod shaped suspension cables, etc.

5.1 Deriving exponent ω as a function of stress range and diameter

Now, in order to be able to derive exponent ω by fracture mechanics, without additional tests, the first step is to develop equation (10) into equation (42). In this equation, the exponent ω is precisely the slope that results from the linear regression of the fatigue damage D , expressed as a function of the relative cycles N/N_f , both on a logarithmic scale. However, fatigue damage D still has to be correlated with the relative cycles N/N_f on a physical/mechanical basis, which is done by the fracture mechanics theory, defining equation (43) as the fatigue damage function that depends on the number of cycles $D(N)$, as in [8]. In this equation a_{th} is the threshold crack size, below which no fatigue occurs, a_c is the critical crack size, beyond which fatigue failure occurs, and $a(N)$ is the actual crack size depending on the number of cycles, passing from a_{th} with $N=0$ to a_c with $N=N_f$. Therefore, the damage function D that is defined in this way passes from 0, when $N=0$, to 1, when $N=N_f$.

$$\log(D) = \omega \cdot \log\left(\frac{N}{N_f}\right) \quad (42)$$

$$D(N) = \frac{a(N) - a_{th}}{a_c - a_{th}} \quad (43)$$

The process to derive the equation that defines the crack size as a function of the number of cycles $a(N)$ starts with the Paris law [28], defined in equation (44), as stated in the seminal work leading to fatigue treatment in the Eurocode [17], with material constants $A=2 \cdot 10^{-13}$ and $m=3$ corresponding to steel [29], and the stress intensity factor defined in (45), yielding equation (46). This approach is suitable for details that have initial crack length above the threshold, normally the case for structural applications, otherwise most fatigue life is expended in crack nucleation and there are more cycles to take into account.

$$\frac{da}{dN} = A \cdot \Delta K^m \quad (44)$$

$$\Delta K = \Delta \sigma \cdot Y(a) \cdot \sqrt{\pi \cdot a} \quad (45)$$

$$\frac{da}{dN} = A \cdot (\Delta \sigma \cdot Y(a) \cdot \sqrt{\pi \cdot a})^m \quad (46)$$

The geometry factor for circular sections $Y(a)$ is taken from [30] and is defined in equation (47), where r is the radius. The threshold crack size a_{th} and the critical crack size a_c [31, 32] are respectively developed, in equations (48) and (49), as functions of K_{th} and K_{IC} , both borrowed from [33, 34] and [35], in the same way as shown in [27].

$$Y(a) = \frac{\frac{1.84}{\pi} \cdot \left[\tan\left(\frac{\pi \cdot a}{4 \cdot r}\right) / \left(\frac{\pi \cdot a}{4 \cdot r}\right) \right]^{0.5}}{\cos\left(\frac{\pi \cdot a}{4 \cdot r}\right)} \cdot \left\{ 0.752 + 2.02 \cdot \left(\frac{a}{2 \cdot r}\right) + 0.37 \cdot \left[1 - \sin\left(\frac{\pi \cdot a}{4 \cdot r}\right) \right]^3 \right\} \quad (47)$$

$$a_{th} = \frac{1}{\pi} \cdot \left(\frac{K_{th}}{Y(a_{th}) \cdot \Delta \sigma_{lim}} \right)^2 \quad (48)$$

$$a_{cr} = \frac{1}{\pi} \cdot \left(\frac{K_{IC}}{Y(a_{cr}) \cdot \sigma_{sup}} \right)^2 \quad (49)$$

Now, after substituting equations (47), (48) and (49) in equation (46) and solving it, the crack size as a function of relative cycles is derived, $a=f(N/N_f)$. Besides, introducing the new found $a=f(N/N_f)$ relationship in equation (43), the damage as function of relative cycles can be derived too, $D=f(N/N_f)$. Finally, substituting the new found damage relationship in equation (42) and finding out ω , the equation (50) is derived.

$$\omega = \frac{\text{Log}[D(N/N_f)]}{\text{Log}(N/N_f)} \quad (50)$$

Taking this into account, in view of equation (50), if the damage as a function of relative cycles $D=f(N/N_f)$ is plotted in a double logarithmic scale graph, then the ω is derived by linear regression, as the mean slope of the line passing by the origin. Then, repeating this method for each pair of range $\Delta\sigma$ and diameter Φ values, the exponent values can be obtained, and are summarized in Table 1.

Table 1: Exponent values depending on stress range and diameter $\omega = f(\Delta\sigma, \Phi)$, [20]:

| $\Delta\sigma \backslash \Phi$ | $\Phi 10$ | $\Phi 12$ | $\Phi 16$ | $\Phi 20$ | $\Phi 25$ | $\Phi 32$ |
|--------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| 50 | 3.0857 | 3.2285 | 3.4795 | 3.7045 | 3.9529 | 4.2543 |
| 100 | 3.0444 | 3.1813 | 3.4357 | 3.6100 | 3.8407 | 4.1430 |
| 150 | 2.9418 | 3.0824 | 3.3264 | 3.5587 | 3.7838 | 4.0301 |
| 200 | 2.9190 | 3.0412 | 3.2945 | 3.4464 | 3.6773 | 3.9594 |
| 250 | 2.8704 | 3.0090 | 3.2448 | 3.4000 | 3.6251 | 3.8490 |
| 300 | 2.8513 | 2.9785 | 3.1507 | 3.3624 | 3.5184 | 3.7415 |
| 350 | 2.8187 | 2.9391 | 3.1039 | 3.2588 | 3.4676 | 3.6895 |
| 400 | 2.7785 | 2.8450 | 3.0792 | 3.2177 | 3.4174 | 3.5828 |
| 450 | 2.7552 | 2.8220 | 3.0305 | 3.1833 | 3.3298 | 3.5308 |
| 500 | 2.6640 | 2.7791 | 2.9442 | 3.1328 | 3.2823 | 3.4777 |

Thus, the only remaining task is to define the exponent ω as a function of the diameter and the stress range $\omega=f(\Phi, \Delta\sigma)$. This definition is done through a triple linear regression: the first one obtains, for each diameter, the slope and the constant of the exponent only as a function of the stress range. Then, the second and third respectively calculate the constant and the slope of this last linear equation as a function of the diameter. The resulting equation is presented in (51), the correspondence of which with the discrete values summarized in Table 1 is shown in Fig. 4.

$$\omega(\phi, \Delta\sigma) = -4 \cdot 10^{-5} \cdot \phi \cdot \Delta\sigma - 4.84 \cdot 10^{-4} \cdot \Delta\sigma + 5.4215 \cdot 10^{-2} \cdot \phi + 2.615867 \quad (51)$$

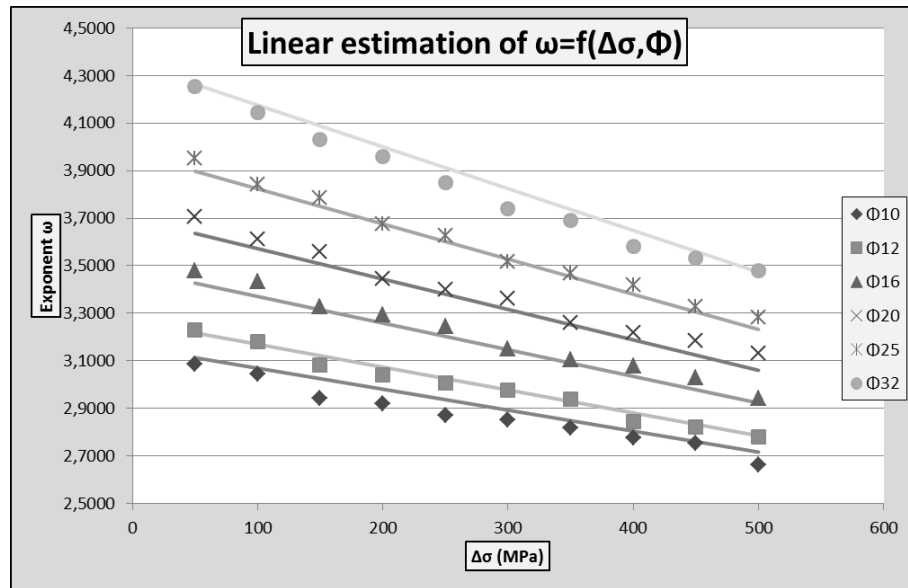


Fig. 4: Correspondence between discrete values of exponent ω summarized in Table 1 and derived from $\omega=f(\Delta\sigma, \Phi)$ (51), [20].

5.2 Calculation of fatigue damage

The steel rod for this calculation corresponds to a steel rebar of 25 mm diameter, subjected to random variable loading of several disordered cycles. The cycle counting is done by the Rainflow method, which yields the following histogram, shown in Fig. 5, ordered by blocks of cycles of varying stress ranges from 50 MPa to 500 MPa.

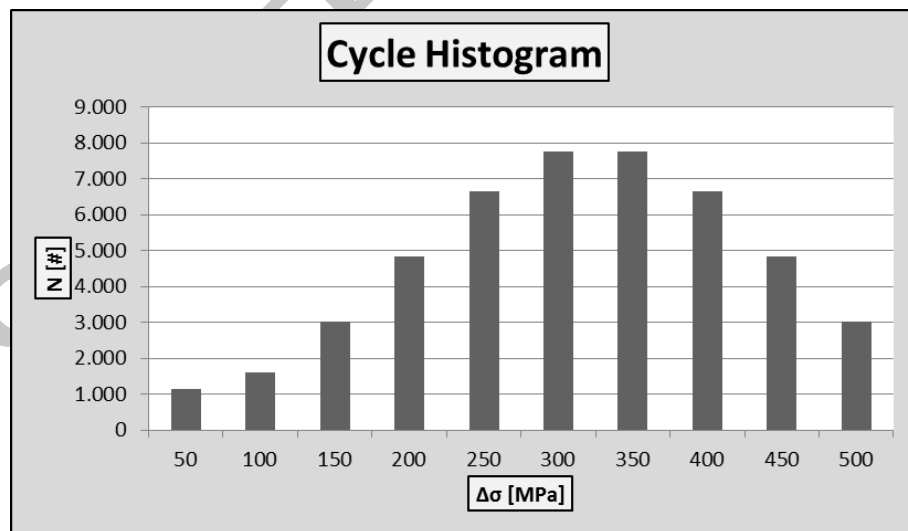


Fig. 5: Cycle histogram of the random process

The next step for the fatigue damage calculation under the Palmgren-Miner linear rule [13, 14] would be to obtain the final number, N_f , or cycles until failure, of the structural element. To do so, the $\Delta\sigma$ - N curves derived from Wohler [21] are employed; in this case, the one specified in [17] for steel rods. This is the original ECCS publication that originated the Structural Eurocodes, based entirely on experimental results. For every stress range level, $\Delta\sigma$, corresponds a characteristic cycles until failure, N_f , that are the amount of cycles with a probability of further survival of 95% with a confidence level of

75%, obtained from all samples tested at each stress range $\Delta\sigma$. The S-N curve is then defined joining all these points. Therefore, taking this detail category we keep the same level of structural reliability. Thus, with a detail category of 100 MPa at $2 \cdot 10^6$ cycles, a constant amplitude fatigue limit (CAFL) of 74 MPa at $5 \cdot 10^6$ cycles, and a cut-off limit of 40 MPa at 10^8 cycles, corresponding to a steel rebar, with slope 1/3 for stress ranges greater than CAFL, slope 1/5 between CAFL and the cut-off limit, and slope zero below the cut-off limit. The CAFL corresponds with the Detail Category, obtained by fatigue testing at several stress range levels, doing several tests on each level. For every stress range level $\Delta\sigma$ corresponds a characteristic cycles until failure N_f , that are the cycles with a probability of further survival of 95% with a confidence level of 75%, obtained from all samples tested at that stress range $\Delta\sigma$. The S-N curve is then defined joining all points. Therefore, taking this detail category we keep the same level of structural reliability.

The actual number of cycles N in each block with a certain stress range $\Delta\sigma$, the cycles until failure N_f and the partial damage N/N_f , are summarized in Table 2.

Table 2: Fatigue damage calculation under the Palmgren-Miner linear rule

| $\Delta\sigma$ | N | N_f | N/N_f |
|----------------|--------|------------------|---------|
| [MPa] | [#] | [#] | [#] |
| 50 | 1.138 | 35.504.106 | 0,0000 |
| 100 | 1.602 | 2.000.000 | 0,0008 |
| 150 | 3.014 | 592.593 | 0,0051 |
| 200 | 4.839 | 250.000 | 0,0194 |
| 250 | 6.636 | 128.000 | 0,0518 |
| 300 | 7.771 | 74.074 | 0,1049 |
| 350 | 7.771 | 46.647 | 0,1666 |
| 400 | 6.636 | 31.250 | 0,2124 |
| 450 | 4.839 | 21.948 | 0,2205 |
| 500 | 3.014 | 16.000 | 0,1883 |
| ΣN | 47.260 | $D=\Sigma N/N_f$ | 0,9698 |

The total damage is then calculated as the summation of all partial damages corresponding to each block. In this case, the direct use of the Palmgren-Miner rule [13, 14] results in damage of 0.9698, a value of less than 1, which predicts the survival of the specimen.

However, the load sequence is disordered, as it is a random load, so there is an acceleration of the damage process that has been neglected by the linear rule approach. The method presented in this paper accounts for this factor, by calculating the exponent ω corresponding to 50 MPa and 500 MPa for a diameter of 25 mm using equation (51) and then the exponents $\omega_1=\omega(500 \text{ MPa}, 25 \text{ mm})$ and $\omega_L=\omega(50 \text{ MPa}, 25 \text{ mm})$ are defined as an input for equation (40), that results in a disorder pushing factor $F_{ED} = 1.073$, representing an increase of 7.3% in fatigue damage. Therefore, the actual maximum fatigue damage, considering the possible sequence effect caused by

disorder, is calculated in (52), yielding a value that is greater than 1, which safely predicts failure.

$$D = F_{ED} \cdot \sum \left(\frac{n_i}{Nf_i} \right) = 1.073 \cdot 0.9698 = 1.0406 \quad (52)$$

6 Conclusions

1. The Palmgren-Miner linear rule, despite its simplicity, is unable to consider the sequence effect, which potentially yields inaccurate predictions in certain cases related to complex random dynamic loading where the fatigue process may be accelerated.
2. The sequence effect on cumulative fatigue damage has then been considered by means of a non-linear rule approach. The upper boundary of the sequence effect has then been determined and used to define a Disorder Pushing Factor F_{ED} , an accelerating coefficient to multiply the application of a plain linear rule for cumulative fatigue damage, thereby defining a new modified linear rule.
3. A case study for a steel rod has been presented, for demonstrative purposes, showing the simplicity of this new methodology. The exponent of the corresponding non-linear rule, and the value of the F_{ED} , in the context of various geometries, and maximum and minimum stress ranges in the sequence for this detail category have been summarized in a table/template. These tables/templates can be easily extended to all detail categories in standard fatigue tables.
4. The new procedure has been applied to a steel rod of 25 mm in diameter, subjected to a random dynamic loading sequence with stress ranges of between 50 and 500 Mpa. The results have revealed an acceleration of the fatigue procedure by up to 7.3%. A fatigue damage prediction of safety using the conventional linear method is therefore shown to be a prediction of failure when using the method that has been advanced in the case study.

7 List of Symbols

| | |
|------------------|--|
| $\Delta\sigma$ | Stress range. |
| $\Delta\sigma_i$ | Stress range of the cycle block i. |
| ω | Exponent of non-linear rule for cumulative damage, normally $f(\Delta\sigma)$. |
| ω_i | Exponent of non-linear rule for cumulative damage of cycle block i with $\Delta\sigma_i$. |
| A | Material constant applied as factor to derive stress intensity factor K. |
| a | Crack size. |
| ac | Critical crack size. |

| | |
|-----------|--|
| ath | Crack threshold. |
| D | Fatigue Damage. |
| D_i | Damage of cycle block i with $\Delta\sigma_i$. |
| D_{ij} | Damage increment passing from a cycle block i with $\Delta\sigma_i$ to a cycle block j with $\Delta\sigma_j$. |
| FED | Disorder Pushing Factor. |
| Jij | Jump from curve i to curve j. |
| L | Ordinal of last block of cycles. |
| m | Material constant applied as exponent to derive stress intensity factor K. |
| N, n | Number of cycles. |
| N_f | Number of cycles until failure at same $\Delta\sigma$. |
| N_{fi} | Number of cycles until failure with stress range $\Delta\sigma_i$. |
| $N_{0,i}$ | Equivalent number of cycles of $\Delta\sigma_i$ applied before cycle block i. |
| n_i | Number of cycles in cycle block i. |
| K | Stress intensity factor. |
| Y(a) | Geometry factor, depending on specimen geometry. |

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Highlights

- A review of sequence effect in fatigue by linear and non-linear rule approaches
- Assessment of non-linear approach suitability for natural random loadings analysis
- Determination of accelerating factor for equivalent damage by linear-rule approach
- Linear-rule suitability for natural random loads considering structural reliability
- Simplicity and suitability demo case with a representative structural element.